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MaaS: Dynamic Reliability Methods in a Clustered-Task Server

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Introduction and Background

- Mathematician, currently at Dell EMC
- Research Interests: Probability Theory, Queuing Theory, Reliability Theory, Stochastic Analysis
- Hobbies: Swimming away from my problems and stockpiling bacon

Why should you care?

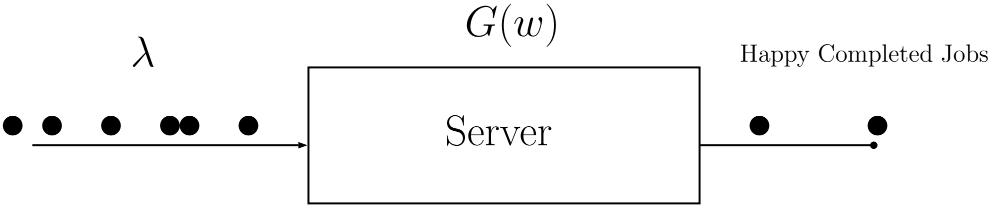
Simplicity is a prerequisite for reliability. – Edsger Dijkstra

- server downtime is costly and makes people angry
- traffic is not steady, and workload is not constant

Goals

- Walk through an overview of mathematical solutions for server reliability
- Leave the cloud and discuss how this is useful to you

A Mathematician's View of a Server



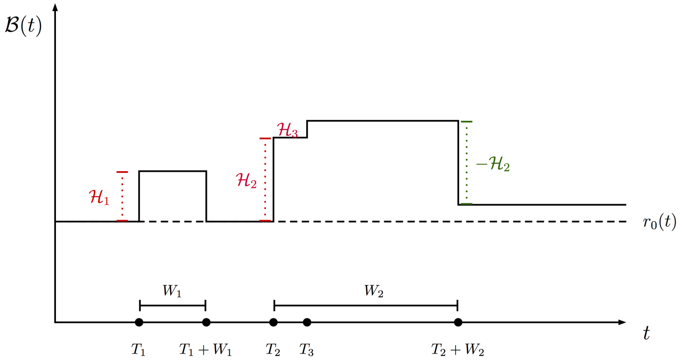
Let’s Talk Reliability

Let Y be the random variable representing the lifetime of a server.

- Survival Function: $S_Y(t) := P(Y > t)$
- Hazard Function - instantaneous rate of failure:

$$h(t) = \lim_{dt \to 0} \frac{P(t \leq Y \leq t + dt)}{dt}$$

Throwing in some randomness



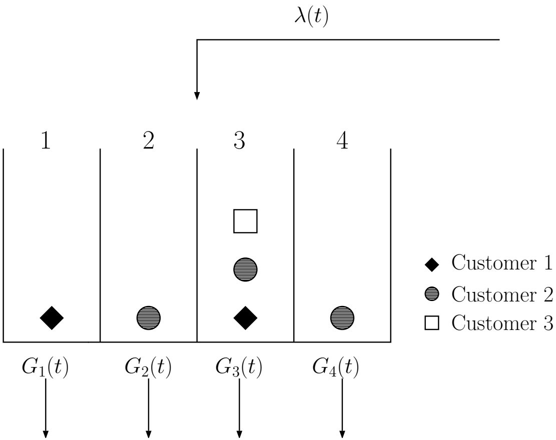
$$\mathscr{B}(t) = r_0(t) + \sum_{j=1}^{N(t)} \mathcal{H}_j \mathbb{1}(T_j < t \leq T_j + W_j), \quad t \geq 0$$

Survival Function for a Single Server

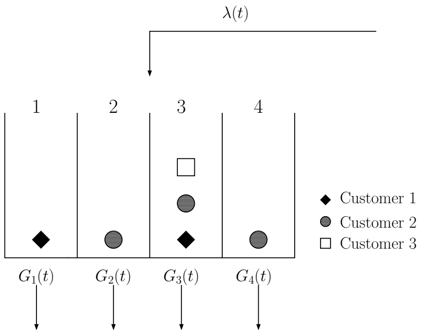
- random arrival times - even the arrival rate can depend on time (non-homogenous Poisson process)
- random service times with any service time distribution you want
- random stress with any distribution you want, continuous or discrete

$$S_Y(t) = \exp\left(-\int_0^t r_0(s)ds\right)\exp\left(-E_{\mathcal{H}}\left[\mathcal{H}\int_0^t e^{-\mathcal{H}w} m(t-w)\bar{G}_W(w)dw\right]\right)$$

Changing up the model- Clustered Task Server



So many options



- independent channel selection
- dependent channel selection - constant, temporal, and random

Independent Channel Selection

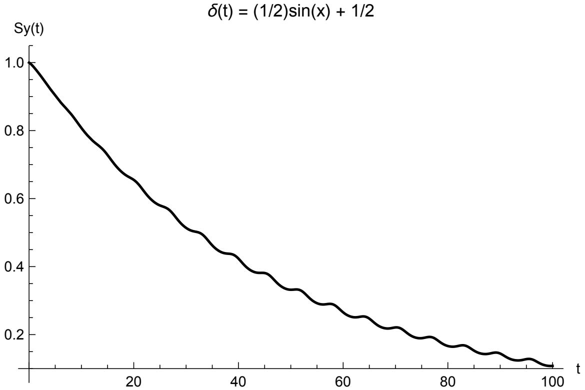
- $p = p_1 = p_2 = \ldots = p_K$ - channel selection probability
- $G_1(w), \ldots G_K(w)$ - service time distributions. $G_W(w) = max_i G_i(w)$
- ηN = stress to the server, N - random number of channels selected
- $\eta N \sim \text{Binomial}(K, p)$

$$S_Y(t) = \bar{F}_0(t) \exp\left(-K\eta\left[e^{-\eta t}(1-p+pe^{-\eta t})^{K-1} - p(1-p)^{K-1}\right] \times \int_0^t m(t-w)\bar{G}_W(w)dw\right)$$

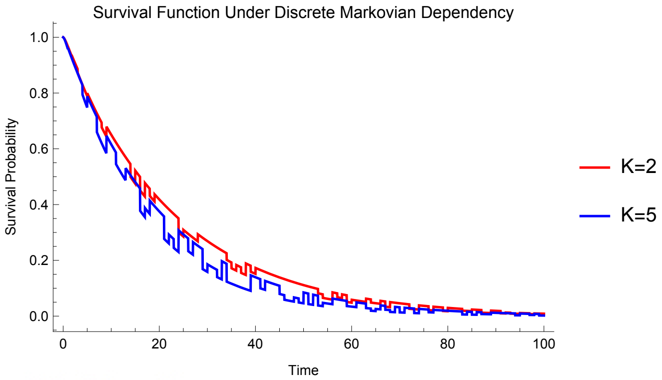
Dependent Channel Selection

- Whether or not the first channel was selected affects the probability of all the other being selected
- Called *first-kind dependency*. (Other kinds detailed on The Math Citadel)
- introduce a *dependency coefficient* $\delta \in [0, 1]$
- Can still give the survival function in closed form

Now let's relax even δ



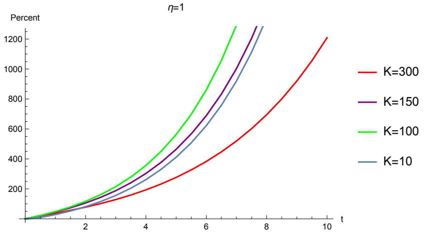
Now let's relax even δ



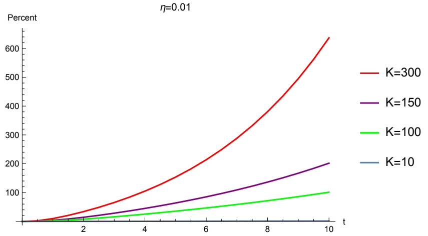
- $\lambda = \eta = 1$, $r_0 \equiv 0.01$, $p = 3/4$, $\bar{G}_W(w) = e^{-w}$
- δ is a 3 state Markov Chain.
- $\delta \in \{1/10, 1/2, 9/10\}$
- Transition Matrix

$$\mathbb{P} = \begin{bmatrix} 1/3 & 1/3 & 1/3 \\ 0 & 2/3 & 1/3 \\ 1/3 & 0 & 2/3 \end{bmatrix}$$

Why do I even care about dependence? Isn't assuming independence enough?



(a) $\eta = 1$



(b) $\eta = 0.01$

Figure: Percent Difference in Independent and Dependent Channel Survival Functions

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Fantastic, now what?

- Let’s do something with them and talk more.
- What kind of applications do you see in this?

My questions for you to think about

- How would you define "stress" to a server?
- What metric do you use to measure traffic?
- Do you collect this data?
- Are you interested in the notion of network reliability? That is, a network built from servers that behave any of these ways?

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